3) $\frac{1}{\cos^4 x}=\frac{1}{3}\left[f'(x)+\frac{2}{\cos^2 x}\right]$

d'où $\int_0^{\frac{\pi}{4}}\frac{1}{\cos^4 x}dx=\frac{1}{3}\left(\int_0^{\frac{\pi}{4}}f'(x)dx+2\int_0^{\frac{\pi}{4}}\frac{1}{\cos^2 x}dx\right)$

$=\frac{1}{3}\left(\left[f(x)\right]_0^{\frac{\pi}{4}}+2\times 1\right)=\frac{1}{3}\left(f(\frac{\pi}{4})-f(0)+2\right)=\frac{4}{3}$

EX 13 :

$f(x)=\frac{1}{\cos x}$; $x\in\left[0,\frac{\pi}{4}\right]$

1) $0\le x\le\frac{\pi}{4}\Rightarrow\frac{\sqrt{2}}{2}\le\cos x\le 1$

$\Rightarrow 1\le\frac{1}{\cos x}\le\sqrt{2}\Rightarrow 1\le f(x)\le\sqrt{2}$

2) $1\le f(x)\le\sqrt{2}$; $\forall x\in\left[0,\frac{\pi}{4}\right]$

d'où : $\int_0^{\frac{\pi}{4}}dx\le\int_0^{\frac{\pi}{4}}\frac{dx}{\cos x}\le\int_0^{\frac{\pi}{4}}\sqrt{2}$

$\Rightarrow(\frac{\pi}{4}-0)\le\int_0^{\frac{\pi}{4}}\frac{dx}{\cos x}\le\sqrt{2}(\frac{\pi}{4}-0)$

$\Rightarrow\frac{\pi}{4}\le\int_0^{\frac{\pi}{4}}\frac{dx}{\cos x}\le\frac{\pi.\sqrt{2}}{4}$

EX 14 :

$I=\int_{\frac{\pi}{4}}^{\pi}\frac{\sin x}{1+x^2}dx$

1) a) $\frac{\pi}{2}\le x\le\pi\Rightarrow(\frac{\pi}{2})^2\le x^2\le\pi^2$

$\Rightarrow 1+(\frac{\pi}{2})^2\le 1+x^2\le 1+\pi^2$

$\Rightarrow\frac{1}{1+\pi^2}\le\frac{1}{1+x^2}\le\frac{1}{1+(\frac{\pi}{2})^2}$

on a aussi : $0\le\sin x\le 1$

d'où $0\le\frac{\sin x}{1+x^2}\le\frac{1}{1+(\frac{\pi}{2})^2}$

b) $0\le\frac{\sin x}{1+x^2}\le\frac{1}{1+(\frac{\pi}{2})^2}$; $\forall x\in\left[\frac{\pi}{2},\pi\right]$

d'où : $0\le\int_{\frac{\pi}{2}}^{\pi}\frac{\sin x}{1+x^2}dx\le\int_{\frac{\pi}{2}}^{\pi}\frac{1}{1+(\frac{\pi}{2})^2}dx$

$\Rightarrow 0\le\int_{\frac{\pi}{2}}^{\pi}\frac{\sin x}{1+x^2}dx\le\frac{\pi}{2(1+(\frac{\pi}{2})^2)}$

d'aprés a) : $\frac{1}{1+\pi^2}\le\frac{1}{1+x^2}\le\frac{1}{1+(\frac{\pi}{2})^2}$ $\forall x\in\left[\frac{\pi}{2},\pi\right]$

d'où $\frac{\sin x}{1+\pi^2}\le\frac{\sin x}{1+x^2}\le\frac{\sin x}{1+(\frac{\pi}{2})^2}$ car $\sin x\ge 0$

$\int_{\frac{\pi}{2}}^{\pi}\frac{\sin x}{1+\pi^2}dx\le\int_{\frac{\pi}{2}}^{\pi}\frac{\sin x}{1+x^2}dx\le\int_{\frac{\pi}{2}}^{\pi}\frac{\sin x}{1+(\frac{\pi}{2})^2}dx$

$\Rightarrow\frac{1}{1+\pi^2}\int_{\frac{\pi}{2}}^{\pi}\sin x\,dx\le\int_{\frac{\pi}{2}}^{\pi}\frac{\sin x}{1+x^2}dx\le\frac{1}{1+(\frac{\pi}{2})^2}\int_{\frac{\pi}{2}}^{\pi}\sin x\,dx$

$\int_{\frac{\pi}{2}}^{\pi}\sin x\,dx=\left[-\cos x\right]_{\frac{\pi}{2}}^{\pi}=1$

d'où $\frac{1}{1+\pi^2}\le\int_{\frac{\pi}{2}}^{\pi}\frac{\sin x}{1+x^2}dx\le\frac{1}{1+(\frac{\pi}{2})^2}$

Ex 15 :

$f(x)=\frac{1}{\sqrt{1+x^4}}$; $x\in\mathbb{R}_+$

$I=\int_{100}^{1000}f(t).dt$

1) a) pour $x>0$: $1+x^4\ge x^4$

$\Rightarrow\sqrt{1+x^4}\ge x^2\Rightarrow\frac{1}{\sqrt{1+x^4}}\le\frac{1}{x^2}$

$\Rightarrow f(x)\le\frac{1}{x^2}$ pour $x>0$

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